Geometry: 10.1-10.3 Notes

NAME______________________

10.1 Lines and segments that intersect circles___________________________ Date:________________

Define Vocabulary:

Circle – The set of all points in a plane that are equidistant from a given point

Center (of a circle) – The point from which all points on a circle are equidistant

Radius (of a circle) – A segment whose endpoints are the center and any point on a circle

Chord (of a circle) – A segment whose endpoints are on a circle

Diameter – A chord that contains the center of a circle

Secant – A line that intersects a circle in two points

Tangent (of a circle) – A line in the plane of a circle that intersects the circle at exactly one point

Point of tangency – The point at which a tangent line intersects a circle

Tangent circles – Coplanar circles that intersect in one point

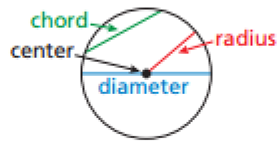
Concentric circles – Coplanar circles that have a common center

Common tangent – A line or segment that is tangent to two coplanar circles

Lines and Segments That Intersect Circles

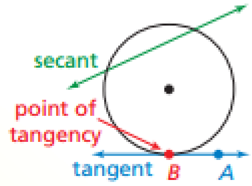
A segment whose endpoints are the center and any point on a circle is a **radius**.

A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.



A **secant** is a line that intersects a circle in two points.

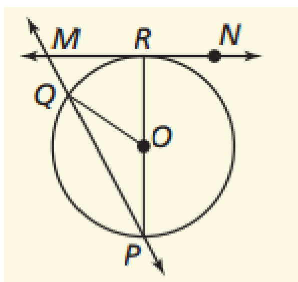
A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the **point of tangency**. The *tangent ray* $\overrightarrow{AB}$ and the *tangent segment* $\overline{AB}$ are also called tangents.



Examples: Identifying Special Segments and Lines

<u>WE DO</u>

Tell whether the line, ray, or segment is best described as a radius, chord, diameter, secant, or tangent of ⊙O.



a. $\overline{PR}$

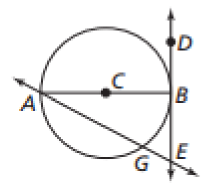
b. $\overleftrightarrow{MN}$

c. $\overleftrightarrow{PQ}$

d. $\overline{QO}$

<u>YOU DO</u>

Tell whether the line, ray, or segment is best described as a radius, chord, diameter, secant, or tangent of ⊙C.

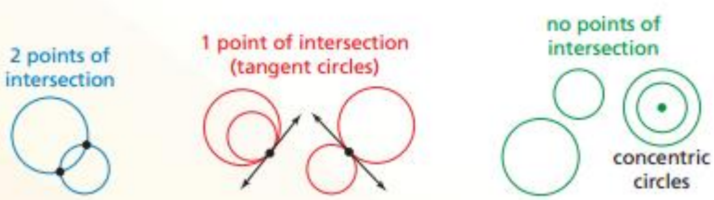


a. $\overline{AG}$

b. $\overline{CB}$

Coplanar Circles and Common Tangents

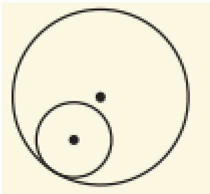
In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric circles**.



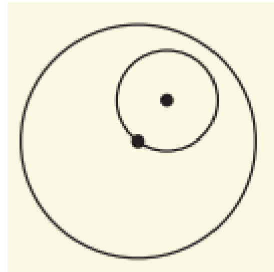
A line or segment that is tangent to two coplanar circles is called a **common tangent**. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

Examples: Tell how many common tangents the circles have and draw them.

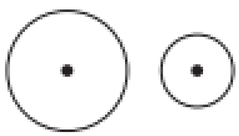
WE DO



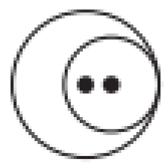
YOU DO



WE DO

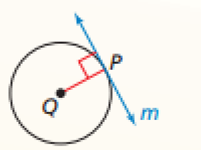


YOU DO



Theorem 10.1 Tangent Line to Circle Theorem

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

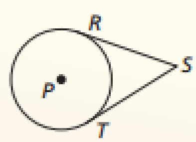


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof Ex. 47, p. 536

Theorem 10.2 External Tangent Congruence Theorem

Tangent segments from a common external point are congruent.



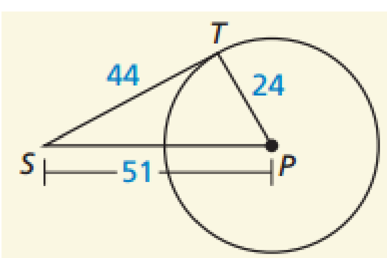
If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof Ex. 46, p. 536

Examples: Verifying a Tangent to a Circle

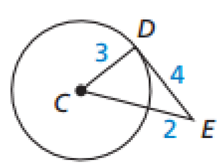
WE DO

Is $\overline{ST}$ tangent to $\odot$P?



YOU DO

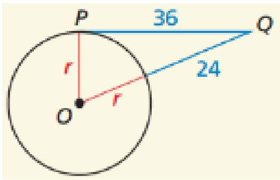
Is $\overline{DE}$ tangent to $\odot$C?



Examples: Finding the Radius of a Circle

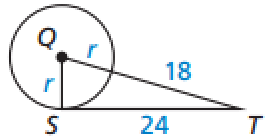
WE DO

In the diagram, point P is a point of tangency. Find the radius r of ⊙O.



YOU DO

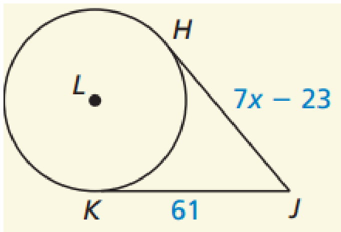
$\overline{ST}$ is tangent to ⊙Q. Find the radius of ⊙Q.



Examples: Using Properties of Tangents

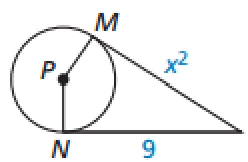
WE DO

$\overline{JH}$ is tangent to ⊙*L* at *H*, and $\overline{JK}$ is tangent to ⊙*L* at *K*. Find the value of *x*.



YOU DO

Points *M* and *N* are points of tangency. Find the value(s) of *x*.



Assignment	

10.2 Finding Arc Measures___________________ __________________________ Date:____________

Define Vocabulary:

Central angle (of a circle) – An angle whose vertex is the center of a circle

Minor arc – An arc with a measure less than 180°

Major arc – An arc with a measure greater than 180°

Semicircle – An arc with endpoints that are the endpoints of a diameter

Measure of a minor arc – The measure of a minor arc's central angle

Measure of a major arc – The measure of a major arc's central angle

Adjacent arcs – Arcs of a circle that have exactly one point in common

Congruent circles – Circles that can be mapped onto each other by a rigid motion or a composition of rigid motions

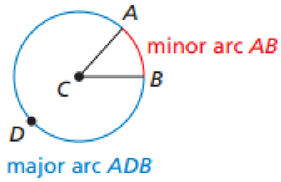
Congruent arcs – Arcs that have the same measure and are of the same circle or of congruent circles

Similar arcs – Arcs that have the same measure

Finding Arc Measures

A **central angle** of a circle is an angle whose vertex is the center of the circle. In the diagram, $\angle ACB$ is a central angle of $\odot C$.

If $m\angle ACB$ is less than 180°, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a **minor arc** with endpoints A and B. The points on $\odot C$ that do not lie on the minor arc AB form a **major arc** with endpoints A and B. A **semicircle** is an arc with endpoints that are the endpoints of a diameter.

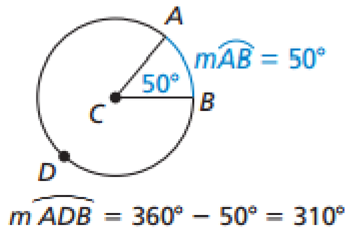


Minor arcs are named by their endpoints. The minor arc associated with $\angle ACB$ is named $\overset{\frown}{AB}$. Major arcs and semicircles are named by their endpoints and a point on the arc. The major arc associated with $\angle ACB$ can be named $\overset{\frown}{ADB}$.

Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\overset{\frown}{AB}$ is read as "the measure of arc AB."

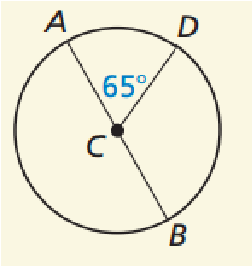
The measure of the entire circle is 360°. The **measure of a major arc** is the difference of 360° and the measure of the related minor arc. The measure of a semicircle is 180°.



Examples: Finding Measures of Arcs

<u>WE DO</u>

Find the measure of each arc of $\odot$C, where $\overline{AB}$ is a diameter.



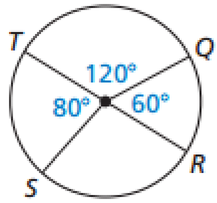
a. $\overset{\frown}{AD}$

b. $\overset{\frown}{DAB}$

c. $\overset{\frown}{BDA}$

<u>YOU DO</u>

Identify the given arc as a major arc, minor arc, or semicircle. Then find the measure of the arc.

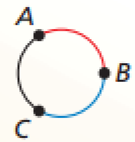


a. $\overset{\frown}{TQ}$

b. $\overset{\frown}{TS}$

Postulate 10.1 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$

Examples: Using the Arc Addition Postulate

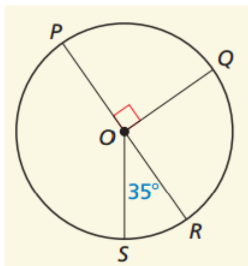
WE DO

Find the measure of each arc.

a. $\widehat{SQ}$

b. $\widehat{RPQ}$

c. $\widehat{PRS}$



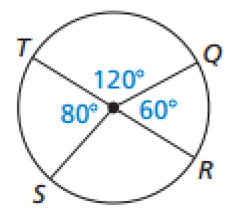
YOU DO

Identify the given arc as a major arc, minor arc, or semicircle. Then find the measure of the arc.

a. $\widehat{QRT}$

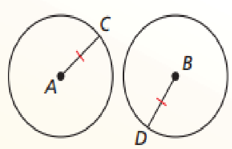
b. $\widehat{TQR}$

c, $\widehat{QS}$



Theorem 10.3 Congruent Circles Theorem

Two circles are congruent circles if and only if they have the same radius.

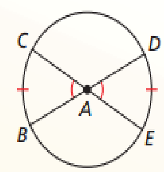


Proof Ex. 35, p. 544

$\odot A \cong \odot B$ if and only if $\overline{AC} \cong \overline{BD}$.

Theorem 10.4 Congruent Central Angles Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.



$\widehat{BC} \cong \widehat{DE}$ if and only if $\angle BAC \cong \angle DAE$.

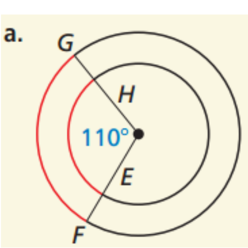
Proof Ex. 37, p. 544

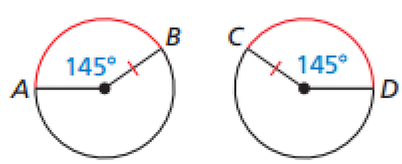
Examples: Identifying Congruent Arcs

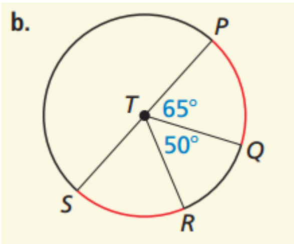
WE DO

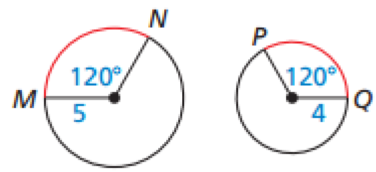
YOU DO

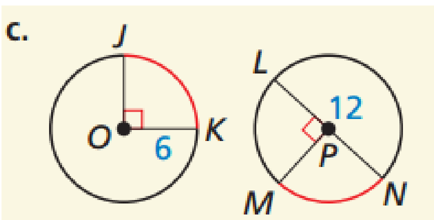
Tell whether the red arcs are congruent. Explain why or why not.











Theorem 10.5 Similar Circles Theorem

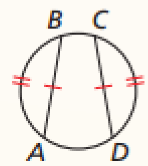
All circles are similar.

Proof p. 541; Ex. 33, p. 544

Assignment	

Theorem 10.6 Congruent Corresponding Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

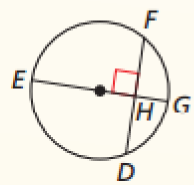


Proof Ex. 19, p. 550

$\widehat{AB} \cong \widehat{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

Theorem 10.7 Perpendicular Chord Bisector Theorem

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

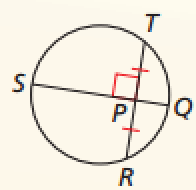


Proof Ex. 22, p. 550

If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\widehat{GD} \cong \widehat{GF}$.

Theorem 10.8 Perpendicular Chord Bisector Converse

If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.



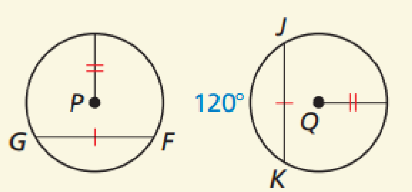
Proof Ex. 23, p. 550

If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

Examples: Using Congruent Chords to Find an Arc Measure

WE DO

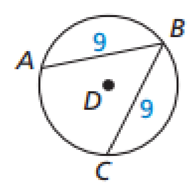
In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\widehat{JK} = 120°$. Find $m\widehat{FG}$.



YOU DO

1. If $m\widehat{AB} = 110°$, find $m\widehat{BC}$.

2. If $m\widehat{AC} = 150°$, find $m\widehat{AB}$.

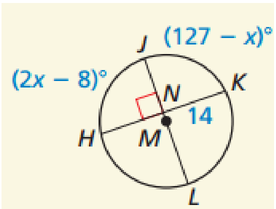


Examples: Using a Diameter

<u>WE DO</u>

a. Find KH

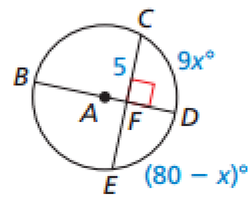
b. Find $m\widehat{HLK}$



<u>YOU DO</u>

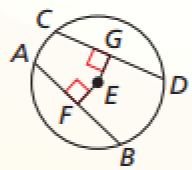
a. Find *CE*

b. Find $m\widehat{CE}$



Theorem 10.9 Equidistant Chords Theorem

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



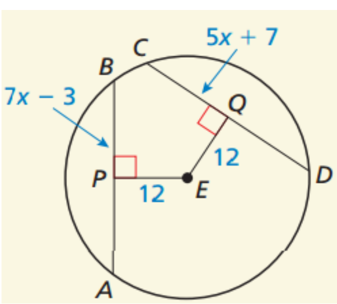
Proof Ex. 25, p. 550

$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

Examples: Using Congruent Chords to Find a Circle's Radius

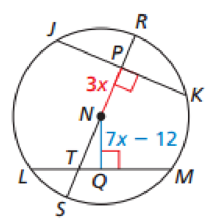
<u>WE DO</u>

In the diagram, $EP = EQ = 12$, $CD = 5x + 7$, and $AB = 7x - 3$. Find the radius of ⊙E



<u>YOU DO</u>

In the diagram, $JK = LM = 24$, $NP = 3x$, and $NQ = 7x - 12$. Find the radius of ⊙N.



Assignment	